(1 pt. BONUS) 1. Given that:

$$b_1 = \frac{\sum[(x_i - \bar{x})(y_i - \bar{y})]}{\sum(x_i - \bar{x})^2} = \sum a_i y_i$$

$$\sigma_{b_1} = \sigma\sqrt{\sum a_i^2} = \frac{\sigma}{\sqrt{\sum(x_i - \bar{x})^2}} = \frac{\sigma}{\sqrt{S_{xx}}}$$

Show that

$$\sqrt{\sum a_i^2} = \frac{1}{\sqrt{S_{XX}}}$$

(12 pts.) 2. An investigative reporter believes that certain automobile service stations that offer state vehicle inspections routinely charge for unnecessary repair work. Preliminary data suggest that the cost of the repair work may be related to the age of the car. A random sample of automobiles inspected at these stations was obtained, and the age (in years) along with the cost of the repairs (in dollars) were recorded for each vehicle. You may assume that all of he assumptions for regression are valid. The summary data is given below:

$n = 15$, $S_{XX} = 82.89$, $S_{YY} = 3{,}848{,}000$, $S_{XY} = 7686$, $\bar{x} = 4.887$, $\bar{y} = 823.3$

(1 pt.) a) What is the explanatory variable? What is the response variable?
(2 pts.) b) Find the estimated regression line.
(1 pt.) c) Find and interpret the 95% confidence interval for the slope.
(6 pts.) d) Conduct a hypothesis test (t-test) to determine if there is an association between age of the car and repair cost.
(1 pt.) e) Find the sample correlation coefficient.
(1 pt.) f) Using your results for c), d), and e), do you believe the reporter's claim? Justify your answer.

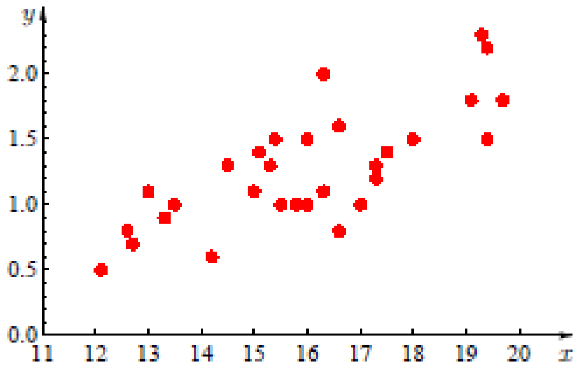
(8 pts.) 3. The temperature of the upper layer of ocean water is affected by sunlight and wind. There is often a very sharp difference in temperature between the surface zone and the more stationary deep zone. The thermocline layer marks the abrupt drop-off in temperature. The following information was obtained in a study of temperature (*x*, measured in °C) versus depth (*y*, measured in meters) above the thermocline layer in the Mediterranean Sea. You may assume that the assumptions for inference are valid.

Source of variation	Sum of squares	Degrees of Freedom	Mean square
Regression	108.54	1	108.34
Error	78.06	6	13.01
Total	186.60	7	

$\hat{y} = 199.32 - 0.0840\,x$

(6 pts.) a) Conduct a model utility hypothesis test (F-test) to determine if there is an association between temperature and depth at a 5% significance level.
(1 pt.) b) From the information provided, find the sample correlation coefficient.
(1 pt.) c) Do you think that there is an association between temperature and depth? Please explain your answer.

(3 pts.) 4. Crimini mushrooms are more common than white mushrooms, and they contain a high amount of copper, which is an essential element according to the U.S. Food and Drug Administration. A study was conducted to determine whether the weight of a mushroom is linearly related to the amount of copper it contains. A random sample of crimini mushrooms was obtained, and the weight (in grams) and the total copper content (in mg) was measured for each. The scatterplot of copper content (y) vs. weight (x) is shown below.



(1 pt.) a) What it be appropriate to use this study to predict the copper content from the mushroom that weighs 27 g? Please explain your answer.
(1 pt.) b) What it be appropriate to use this study to predict the copper content from the mushroom that weighs 7 g? Please explain your answer.
(1 pt.) c) What it be appropriate to use this study to predict the copper content from the mushroom that weighs 17 g? Please explain your answer.

(4 pts.) 5. A study was conducted to investigate the relationship between asthma prevalence and annual rainfall in countries around the world. A random sample of 11 countries was obtained. The total annual rainfall (*x*, in mm) was recorded for each country, as well as the percentage of adults, 22–44 years old, treated for asthma (*y*). The following summary statistics were reported.

$b_0 = 3.5955$ $b_1 = 0.0021$ $\bar{x} = 792.636$ MSE = 5.3851 $S_{XX} = 178{,}661$

(2 pts.) a) Find and interpret a 95% confidence interval for the true mean observed asthma prevalence if the total annual rainfall is 1000 mm. Comment on anything odd about this interval.
(2 pts.) b) Find and interpret a 95% prediction interval for an observed asthma prevalence if the total annual rainfall is 1000 mm. Comment on anything odd about this interval.